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The rainbow connection number of the enhanced power graph of a finite group

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Abstract

Let G be a finite group. The enhanced power graph Γ_G^e of G is the graph with vertex set G and two distinct vertices are adjacent if they generate a cyclic subgroup of G. In this article, we calculate the rainbow connection number of Γ_G^e.

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1. Introduction

Let G be a finite group. The power graph of G, denoted Γ_G, is the graph whose vertex set is G and two distinct vertices are adjacent if one is a power of the other. In [1] the authors found that the power graph is contained in the non-commuting graph and, they asked about how much the graphs are closer, and then, they defined the *enhanced power graph* of a finite group. The enhanced power graph of G, denoted Γ_G^e, is the graph whose vertex set is the group G and two distinct vertices $x, y \in V(\Gamma_G^e)$ are adjacent if $x, y \in \langle z \rangle$ for some $z \in G$. In recent years, the study of enhanced power graphs has been growing, see, for example, "The metric dimension of the enhanced power graph of a finite group" and "A study of enhanced power graphs of finite groups" will appear in Journal of Algebra and Its Applications.

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In 2006, Chartrand, Johns, McKean and Zhang [8] introduced the concept of rainbow connection of graphs. This concept was motivated by communication of information between government agencies of the United States after the September 11, 2001, terrorist attacks. The situation that helps to unravel this issue about communications has the following graph-theoretic model. Let Γ be a nontrivial connected graph, with vertex set $V(\Gamma)$ and edge set $E(\Gamma)$. For $k \in \mathbb{N}$, we define an edge–coloring as $\zeta : E(\Gamma) \to \{1, \ldots, k\}$, where adjacent edges may have the same color. A path P is a *rainbow* if no two edges have the same color. An edge–colored graph Γ is *rainbow connected* if every two distinct vertices are connected by a rainbow. An edge-coloring under which Γ is rainbow connected is called *rainbow coloring*. The *rainbow connection number of* Γ, denoted by $rc(\Gamma)$, is the smallest number of colors that are needed in order to make Γ rainbow connected.

We will apply the idea of calculating the rainbow connected number of enhanced power graph through the graphs such as was carried out by the authors from [10] about the power graph, with the set of maximal involutions; denoted by M_G the set of maximal involutions of G, whose important theorems we can summarize in the following:

Theorem 1.1. *Let G be a finite group of order at least* 3 *and let* $M_G \neq \emptyset$. *Then*

$$rc(\Gamma_G) = \begin{cases} 3, & \text{if } 1 \leq |M_G| \leq 2, \\ |M_G|, & \text{if } |M_G| \geq 3. \end{cases}$$

If $|M_G| = \emptyset$, *let G be a finite group*

1. *If G is cyclic, then* $rc(\Gamma_G) = \begin{cases} 1, & \text{if } |G| \text{ is a prime power}, \\ 2, & \text{otherwise.} \end{cases}$
2. *If G is noncyclic, then* $rc(\Gamma_G) = 2$ *or* 3.

In this paper we compute the rainbow connection number of Γ_G^e and characterize it in terms of independence cyclic set, whose particular case is maximal involution. This paper is organized as follows. In section 2 we give some definitions and properties about rainbow connection number and we describe a way to guarantee a coloring for enchanced power graphs. In section 3 we summarize the main theorems for determining Γ_G^e.

2. Definitions and properties

The next proposition follows from the definition of an enhanced power graph.

Proposition 2.1. $rc(\Gamma_G^e) = 1$ *if and only if* Γ_G^e *is complete if and only if G is cyclic.*

Definition 1. Let $Max_G = \{x_1, \ldots, x_m\}$ be an *essential cyclic set* if

1. $\langle x_i \rangle \neq \langle x_j \rangle$ for $i \neq j$,
2. Each x_i is a maximal cyclic subgroup.

Therefore Proposition 2.1 can be rewritten as follows

Proposition 2.2. $|Max_G| = 1$ *if and only if* G *is a cyclic group if and only if* $rc(\Gamma_G^e) = 1$

Proposition 2.3. *If* $|Max_G| = 2$*, then* $rc(\Gamma_G^e) = 2$

Proof. Observe that if $Max_G = \{x_1, x_2\}$, $1 \neq g$, and $g \in \langle x_1 \rangle \cap \langle x_2 \rangle$, then $\langle g \rangle = \langle x_1 \rangle$ or $\langle g \rangle = \langle x_2 \rangle$. Without loss of generality suppose that $\langle g \rangle = \langle x_1 \rangle$, then $\langle x_1 \rangle = \langle g \rangle \subset \langle x_2 \rangle$, but that is a contradiction because Max_G is an essential cyclic set. Therefore $|\langle x_1 \rangle \cap \langle x_2 \rangle| = 1$. Since Γ_G^e is not complete, we have $rc(\Gamma_G^e) \geq 2$, then,

$$\begin{array}{rcl} E_1 & = & \big\{\{a,b\} | a, b \in \langle x_1 \rangle\big\} \\ E_2 & = & \big\{\{a,b\} | a, b \in \langle x_2 \rangle\big\} \end{array}$$

We notice that the only path between x_{1_j} and x_{2_i} for all $x_{1_j} \in \langle x_1 \rangle$ and $x_{2_i} \in \langle x_2 \rangle$ is (x_{1_j}, e, x_{2_i}), then, the 2-coloring is given by $\zeta : E(G) \longrightarrow \{1, 2\}$ with $f \mapsto i$, if $f \in E_i$ is a rainbow 2-coloring of Γ_G^e. □

Definition 2. We define the *independence cyclic set of* Max_G, denoted by $ics(G)$, as

$$ics(G) = \{x_i \in Max_G | \langle x_i \rangle \cap \langle x_j \rangle = e \text{ for } i \neq j\}$$

The independence cyclic number of Max_G, denoted by $icn(G)$, is $icn(G) = |ics(G)|$.

Remark 2.1. We note that

$$M_G \subseteq ics(G) \subseteq Max_G.$$

Proposition 2.4. *If* $|Max_G| = 3$*, then*

$$rc(\Gamma_G^e) = \begin{cases} 2, & \text{if } icn(G) = 1, \\ 3, & \text{if } icn(G) = 3. \end{cases}$$

Proof. Let $Max_G = \{x_1, x_2, x_3\}$ be an essential cyclic set.

Remark 2.2. We do not need to be concise with the path with both vertex in $\langle x_i \rangle$ for some i, because with one color, we can coloring this path. The difficulty is when both vertex are in different $\langle x_i \rangle$.

Case 1. $\boldsymbol{icn(G) = 1}$

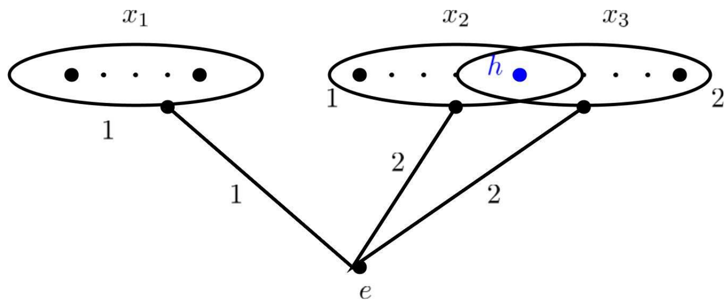
Without loss of generality we suppose $\langle x_1 \rangle \cap \langle x_2 \rangle = e = \langle x_1 \rangle \cap \langle x_3 \rangle$ and $\langle x_2 \rangle \cap \langle x_3 \rangle \neq e$. Since G is not cyclic group, then $rc(\Gamma_{G^e}) \geq 2$. Let $h \in \langle x_2 \rangle \cap \langle x_3 \rangle$ with $h \neq e$, thus

$$\begin{array}{rcl} E_1 & = & \big\{\{a,b\} | \{a,b\} \subset \langle x_1 \rangle\big\} \bigcup \big\{\{a,b\} | \{a,b\} \subset \langle x_2 \rangle \text{ with } a, b \neq e\big\} \\ E_2 & = & \big\{\{e,g\} | g \in \langle x_2 \rangle \cup \langle x_3 \rangle\big\} \bigcup \big\{\{a,b\} | a \in \langle x_3 \rangle \setminus \langle x_2 \rangle, \quad b \in \langle x_2 \rangle \cap \langle x_3 \rangle,\, b \neq e\big\} \end{array}$$

In particular $\{h, g\} \in E_2$ for all $g \in \langle x_3 \rangle \setminus \langle x_2 \rangle$. Then, we will give a 2-coloring to Γ_G^e:

$$\begin{array}{rcl} \zeta : E(G) & \longrightarrow & \{1,2\} \\ f & \mapsto & i \end{array} \text{ if } i \in E_i \tag{1}$$

Case 2. $\boldsymbol{icn(G) = 3}$



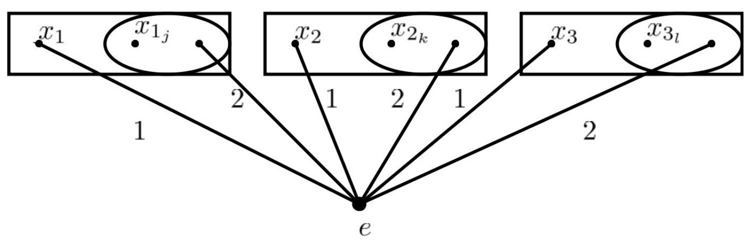
We suppose that $|M_G| = 0$, and without loss of generality $\langle x_i \rangle \cap \langle x_j \rangle = e$ for $1 \leq i < j \leq 3$. We will give a 3-coloring for Γ_{G^e}, with

$$\begin{array}{rcl} E_1 & = & \big\{\{x_i, e\} | i = 1, 2, 3\big\} \\ E_2 & = & \big\{\{e, x_{i_j}\} | x_{i_j} \in \bigcup_{i=1}^{3} \langle x_i \rangle \setminus x_i\big\} \\ E_3 & = & \big\{\{a, b\} | a, b \in \langle x_i \rangle \text{ for } i = 1, 2, 3\big\} \end{array}$$

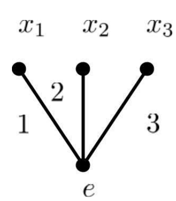
With the coloring

$$\begin{array}{rcl} \zeta : E(G) & \longrightarrow & \{1, 2, 3\} \\ f & \mapsto & i \end{array} \quad \text{if } i \in E_i \tag{2}$$

Now, we suppose that $M_G = Max_G$, then with $E_i = \big\{\{a, b\} | a, b \in \langle x_i \rangle\big\}$. be the edges set,



and the coloring is given like (2). We can not give a 2-coloring for Γ_G^e. We claim that there is a



2-coloring. Let $u \in \langle x_1 \rangle$, $v \in \langle x_2 \rangle$ and $w \in \langle x_3 \rangle$. Then, we have $\zeta(u, e) = 1$ and $\zeta(e, v) = 2$, thus (u, e, v) is the desire rainbow path. Likewise $\zeta(u, e) = 1$ and $\zeta(e, w) = 2$, but for (v, e, w) there is not a rainbow path. □

The Proposition 2.4 lead us to ask what happens if no one of $\langle x_i \rangle$ can be intersected by another $\langle x_j \rangle$ with $i \neq j$ or, what happens if all $\langle x_i \rangle$ are intersected with some common elements. For this, we have the next propositions.

The following proposition is just like [10, Proposition 2.4]

Proposition 2.5. *Let $Max_G = \{x_1, \ldots, x_m\}$ be an essential cyclic set and $M_G = Max_G$. Then $rc(\Gamma_G^e) = m$.*

Proof. For Γ_G^e we will give a m-coloring. For each $i = 1, \ldots, m$ we have

$$E_i(G) = \big\{\{a, b\}|a, b \in \langle x_i\rangle\big\}$$

since for $u \in \langle x_i\rangle$ and $v \in \langle x_j\rangle$ with $i \neq j$ we have one path between them, which is (u, e, v), and the coloring is given by

$$\begin{array}{rcl} \zeta : E(G) & \longrightarrow & \{1, \ldots, m\} \\ f & \mapsto & i \quad \text{if } f \in E_i \end{array}$$

We can see the diagram in figure 1. □

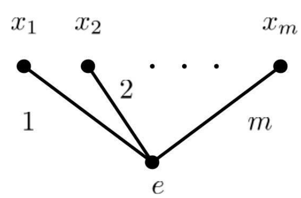


Figure 1. $M_G = Max_G$

Proposition 2.6. *Let $Max_G = \{x_1, \ldots, x_m\}$ be an essential cyclic set with $m \geq 2$, and $h_{i,j} \in \langle x_i\rangle \cap \langle x_j\rangle$ for $1 \leq i < j \leq m$. If $h_{i,j} \neq h_{r,s}$, with $i \neq r$ or $j \neq s$, then $rc(\Gamma_G^e) = 2$.*

Proof. By Remark 2.2 we only give the coloring for x_i and x_j such that $i < j$. We fix

$$E_1(G) = \big\{\{a, h_{i,j}\}|a \in \langle x_i\rangle \setminus \langle x_j\rangle\big\} \bigcup \big\{\{a, b\}|a, b \in \langle x_i\rangle\big\}$$
$$E_2(G) = \big\{\{b, h_{i,j}\}|b \in \langle x_j\rangle \setminus \langle x_i\rangle\big\} \bigcup \big\{\{a, b\}|a, b \in \langle x_j\rangle\big\}$$

Then, we always have a path for $x_{i_r} \in \langle x_i\rangle$ to $x_{j_s} \in \langle x_j\rangle$ given by $(x_{i_r}, h_{i,j}, x_{j_s})$ with $i < j$, and the coloring is the same given in (1). □

The next definition guarantees the existence of a coloring for Γ_G^e.

Definition 3. An *awning* is a collection $H_1, \ldots, H_{m-1}$ where the following occurs:

1. $H_i = A_i \dot{\bigcup} B_i = \{h_{i,i+1}, \ldots, h_{i,m}\} \subset \langle X_i\rangle$ for $i = 1, \ldots, m-1$
2. For all $i < j$, $h_{i,j} \in \langle x_i\rangle \cap \langle x_j\rangle$
3. For $i < j$ with $j = 2, \ldots, m-1$, if $h_{j,s} = h_{i,r} \in H_j \cap H_i$ ($s \in \{j+1, \ldots, m\}$, and $r \in \{i+1, \ldots, m\}$), the following holds:
   (a) $r = j$, $h_{i,r} \in A_i$, then $h_{j,s} \in B_j$
   (b) $r = j$, $h_{i,r} \in B_i$, then $h_{j,s} \in A_j$

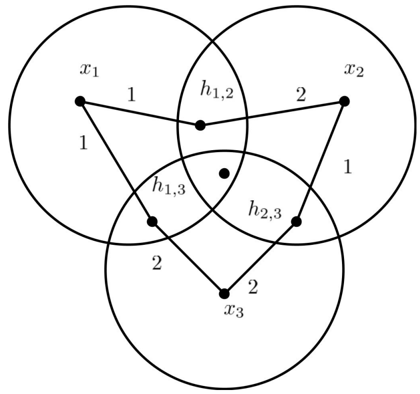


Figure 2. Example for Proposition 2.6 for $m = 3$

(c) $r = s > j$, $h_{i,r} \in A_i$, then $h_{j,r} \in A_j$
(d) $r = s > j$, $h_{i,r} \in B_i$, then $h_{j,r} \in B_j$

Remark 2.3. The case in Proposition 2.5 is a particular case where G does not have an awning. By the definition of awning, we only need $H_i = \{e\}$ for only some i, and no more.

Corollary 2.1. *If G has an awning and $|Max_G| \geq 3$, then $icn(G) \leq 1$. In particular, $|M_G| \leq 1$.*

Proof. Suppose that $icn(G) = 2$, then $H_{i_1} = H_{i_2} = \{e\}$. Hence (x_{i_1}, e, x_{i_2}) is a rainbow path, and (x_{i_1}, e, x_{i_3}) is another rainbow path, but in (x_{i_2}, e, x_{i_j}) we do not have a rainbow path for Γ_G^e. □

Corollary 2.2. *If $|\cap H_i| \geq m - 1$ with $Max_G = m$, then G has an awning.*

Corollary 2.3. *If $|Max_G| = 2$ then $icn(G) = 0$ or 2, and G has an awning.*

Corollary 2.4. *If G has an awning, then $icn(G) = 1$. In particular $|M_G| \leq 1$.*

We notice that the coloring whether we have to M_G or $ics(G)$ does not change, both can be colored by one color. The only difference in M_G is that there is only two elements in the subset of G and, for a set taken of $ics(G)$ there are more than two elements, however, the behaviour in coloring is exactly the same, because, in a set taken of $ics(G)$ all the elements are associated each of them, then, one color is enough for coloring all set.

In the following properties we only consider the set $ics(G)$ unless otherwise indicated.

Proposition 2.7. *If G has an awning, then $rc(\Gamma_G^e) = 2$*

Proof. We will give to Γ_G^e a rainbow 2-coloring, for $1 \leq r < s \leq m$, let:

$$\begin{aligned}
E_{r,s}^1 &= \{\{a, h_{r,s}\} | a \in \langle x_r \rangle \backslash \langle x_s \rangle; h_{r,s} \in A_r\} \\
E_{r,s}^2 &= \{\{b, h_{r,s}\} | b \in \langle x_s \rangle \backslash \langle x_r \rangle; h_{r,s} \in A_r\} \\
E_{r,s}^1 &= \{\{a, h_{r,s}\} | a \in \langle x_r \rangle \backslash \langle x_s \rangle; h_{r,s} \in B_r\} \\
E_{r,s}^2 &= \{\{b, h_{r,s}\} | b \in \langle x_s \rangle \backslash \langle x_r \rangle; h_{r,s} \in B_r\}
\end{aligned}$$

Write $E_1 = \bigcup_{1\leq r<s\leq m} E^1_{r,s}$ and $E_2 = \bigcup_{1\leq r<s\leq m} E^2_{r,s}$ and we define a coloring

$$\begin{array}{rcl} \zeta : E(\Gamma^e_G) & \longrightarrow & \{1,2\} \\ f & \mapsto & i, \quad \text{if } f \in E_i \end{array}$$

We are going to check that this is a 2-coloring for Γ^e_G. We will make a coloring for $j, s-$step. If these edges have been colored in a before step, i.e., if $h_{j,s} = h_{i,r}$ with $i < j$, thus we will have coloring problems with $r = j$ or $r = s$.

For $r = j$ (r=s), for (a)-(d) from Definition 3 we can guarantee in before step we can conserve the coloring and that, not affect us with the 2-coloring that we gave. □

Proposition 2.8. *If $rc(\Gamma^e_G) = 2$, then for any order of Max_G, we have an awning.*

Proof. We have $rc(\Gamma^e_G) = 2$ and suppose $E_1 \dot{\bigcup} E_2 = E$ be the set of edges of Γ^e_G and a 2-coloring given by (1) and let $Max_G = \{x_1, \ldots, x_m\}$ be an independence cyclic set of Γ^e_G, thus there is $h \in \langle x_i\rangle \cap \langle x_j\rangle$ such that $\{x_i, h\} \in E_1$ and $\{h, x_j\} \in E_2$ (or $\{x_i, h\} \in E_2$ and $\{h, x_j\} \in E_1$). We define $h_{i,j} := h$, moreover $H_i := \{h_{i,1}, \ldots, h_{i,m}\} =: A_i \dot{\bigcup} B_i$ such that

$$A_i = \{h_{i,j} | \{x_i, h_{i,j}\} \in E_1\} \text{ and } B_i = \{h_{i,j} | \{x_i, h_{i,j}\} \in E_2\},$$

where $(a) - (b)$ from Definition 3 are met. □

Corollary 2.5. *If G has an awning with any order on Max_G, then for every order, G has an awning.*

Theorem 2.1. *$rc(\Gamma^e_G) = 2$ if and only if G has an awning and G is not cyclic group.*

Proof. By Propositions 2.7 and 2.8 □

By Remark 2.1 we obtain a similar proposition like [10, Lemma 2.2].

Lemma 2.1. *Let $Max_G = \{x_1, \ldots, x_m\}$ be an essential cyclic set. If $M_G \neq \emptyset$, then $|M_G| \leq rc(\Gamma^e_G)$.*

Proof. As in the proof of [10, Lemma 2.2]. □

Proposition 2.9. *Let $Max_G = \{x_1, \ldots, x_m\}$ be an essential cyclic set. If $icn(G) \geq 3$, then $3 \leq rc(\Gamma^e_G)$.*

Proof. Suppose that $|M| = 0$ and $ics(G) = \{x_1, \ldots, x_k\}$ be an independence cyclic set with $k \geq 3$. We can not give a 2-coloring for the graph induced by $\langle x_1\rangle \cup \cdots \cup \langle x_k\rangle$, but we will give a 3-coloring induced by the following edge sets

$$\begin{array}{rcl} E_1 & = & \big\{\{x_i, e\} | i = 1, \ldots, m\big\} \\ E_2 & = & \big\{\{e, x_{i_j}\} | x_{i_j} \in \bigcup_{i=1}^m \langle x_i\rangle \setminus x_i\big\} \\ E_3 & = & \big\{\{a, b\} | a, b \in \langle x_i\rangle \text{ for each } i\big\} \end{array}$$

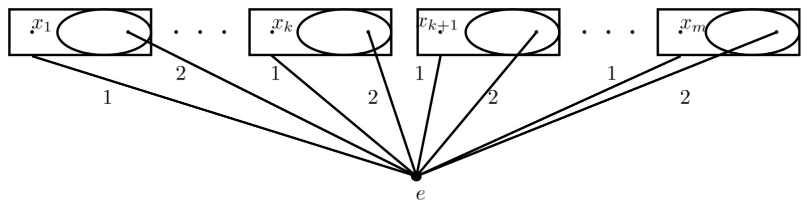


Figure 3. $M_G = \emptyset$

with the rest edges just like Propositions 2.6 and 2.7. Thus the 3-coloring is given by (2).

If $|M_G| \geq 3$ then the edges set is

$$\begin{array}{rcl} E_1 & = & \big\{\{x_i, e\} | i = l+1, \ldots, m\big\} \\ E_2 & = & \big\{\{e, x_{i_j}\} | x_{i_j} \in \bigcup_{i=l+1}^{m} \langle x_i \rangle \setminus x_i\big\} \\ E_3 & = & \big\{\{a, b\} | a, b \in \langle x_i \rangle \text{ for each } i\big\} \\ E_i & = & \big\{\{x_i, e\} | i = 1, \ldots, l\big\} \end{array}$$

and the coloring given by

$$\begin{array}{lcl} \zeta : E(G) & \longrightarrow & \{1, \ldots, l\} \\ f & \mapsto & i \end{array} \quad \text{if } f \in E_i$$

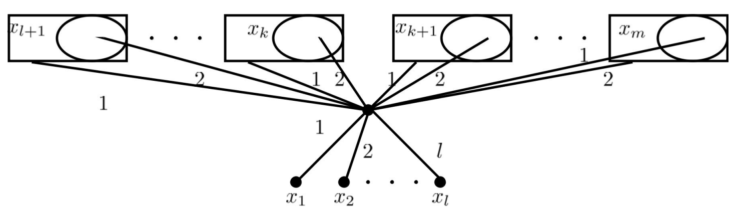


Figure 4. $M_G \neq \emptyset$

□

In particular we have the following

Proposition 2.10. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be a essential cyclic group with* $m \geq 4$ *and* $icn(G) \geq 2$*, then* $3 \leq rc(\Gamma_G^e)$.

Remark 2.4. We have $rc(\Gamma_G^e) \leq rc(\Gamma_G)$ because $E(\Gamma_G) \subseteq E(\Gamma_G^e)$.

3. Main Theorems

In this section we summarize our main theorems as immediate consequences of our previous results and definitions. For instance, by definition of $icn(G)$ and Propositions 2.1, 2.2 and 2.5 it follows the next theorem.

Theorem 3.1. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be an essential cyclic set. If* $icn(G) = 1$ *then* $rc(\Gamma_G^e) = 1$ *if and only if* $m = 1$. *In particular, if* $|M_G| = 1$ *then* $rc(\Gamma_G^e) = 1$ *if and only if* $G \cong \mathbb{Z}_2$.

In a similar way, by Proposition 2.4, Corollary 2.4 and Proposition 2.7 the following theorem holds.

Theorem 3.2. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be an essential cyclic set. If* $icn(G) = 1$ *then* $rc(\Gamma_G^e) = 2$ *if and only if* G *has an awning.*

Accordingly to Proposition 2.9 we get the following result.

Theorem 3.3. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be a essential cyclic set with* $m \geq 3$. *If* $icn(G) = 1$ *then* $rc(\Gamma_G^e) = 3$ *if and only if* G *has not an awning.*

The following theorem holds from Propositions 2.9 and 2.10.

Theorem 3.4. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be a essential cyclic set with* $m \geq 4$. *If* $icn(G) = 2$*, then* $rc(\Gamma_G^e) = 3$.

By Propositions 2.9 and 2.5, we have the next theorem.

Theorem 3.5. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be an essential cyclic set. If* $icn(G) \geq 3$*, then* $rc(\Gamma_G^e) = |M_G|$.

Finally, we have the following theorem.

Theorem 3.6. *Let* $Max_G = \{x_1, \ldots, x_m\}$ *be an essential cyclic set with* $icn(G) = 0$*, then*

$$rc(\Gamma_G^e) = \begin{cases} 1, & \textit{if and only if } G \textit{ is a cyclic group,} \\ 2, & \textit{if and only if } G \textit{ has an awning and } G \textit{ is not cyclic,} \\ 3, & \textit{iff } G \textit{ has not an awning.} \end{cases}$$

Proof. The first case is a consequence of Proposition 2.2. The second case is from Proposition 2.7. The third case is derived from Remark 2.3 and Proposition 2.9. □

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