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On open neighborhood locating-dominating in graphs

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Abstract

A set D of vertices in a graph $G = (V(G), E(G))$ is an open neighborhood locating-dominating set (OLD-set) for G if for every two vertices u, v of $V(G)$ the sets $N(u) \cap D$ and $N(v) \cap D$ are non-empty and different. The open neighborhood locating-dominating number $OLD(G)$ is the minimum cardinality of an OLD-set for G. In this paper, we characterize graphs G of order n with $OLD(G) = 2$, 3, or n and graphs with minimum degree $\delta(G) \geq 2$ that are C_4-free with $OLD(G) = n - 1$.

Keywords: domination; open neighborhood locating-dominating
Mathematics Subject Classification : 05C69

1. Introduction

Various problems in which a graph G models a facility or multiprocessor network involve choosing a vertex set $S \subseteq V(G)$ for the graph $G = (V, E)$, where each $v \in S$ represents a detection device capable of determining if there is an intruder or malfunctioning device in its neighborhood. The open neighborhood of vertex v is $N(v) = \{w \in V(G) : vw \in E(G)\}$, the set of vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$.

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For a locating-dominating set $L \subseteq V(G)$ a detection device at $v \in L$ can determine if an intruder is at v or if the intruder is in $N(v)$, but which element of $N(v)$ can not be determined (that is, if an intruder is at distance zero or at distance one). As introduced in Slater [23, 24, 25], a vertex set $L \subseteq V(G)$ is *locating-dominating* if L dominates $V(G)$ (that is, $\cup_{v \in L} N[v] = V(G)$ so that every intruder can be detected) and for any two vertices x and y in $V(G) - L$ we have $N(x) \cap L \neq N(y) \cap L$. Other works concerning locating-dominating sets include [1, 2, 3, 4, 7, 8, 10, 26, 27]. (Locating sets involving distances larger than one are defined in Slater [22] and Harary and Melter [6].)

If a detection device at v can determine whether there is an intruder in the closed neighborhood $N[v]$, but which vertex location in $N[v]$ cannot be determined, then one is interested in an identifying code, as introduced in Karpovsky, Chakrabarty and Levitin [12]. A vertex set $R \subseteq V(G)$ is an *identifying code* if R dominates $V(G)$ and for any two vertices x and y in $V(G)$ we have $N[x] \cap R \neq N[y] \cap R$. See, for example, [14].

When a detection device at v can determine if an intruder is in $N(v)$ but will not report if the intruder is at v, we are interested in open neighborhood locating-dominating sets as introduced for the k-cubes Q_k by Honkala, Laihonen and Ranto [11] (where they are called "identifying codes with non-transmitting faulty vertices") and for all graphs by Seo and Slater [15]. See also [9, 16, 17, 18, 19, 20, 21]. A vertex set $S \subseteq V(G)$ is an *OLD-set* if S dominates $V(G)$ and for any two vertices x and y in $V(G)$ we have $N(x) \cap S \neq N(y) \cap S$. We note that each strongly-identifying code defined in [11] is both an identifying-code and an OLD-set. Every graph G has a locating-dominating set; G has an identifying code only when no two vertices have the same closed neighborhood; and G has an open neighborhood locating-dominating set only when no two vertices have the same open neighborhood.

The minimum cardinality of an open neighborhood locating-dominating set is denoted here by $OLD(G)$ and an OLD-set S with $OLD(G) = |S|$ is called an *$OLD(G)$-set*. (In particular, every $OLD(G)$-set is an OLD-set for G.) Lobstein [13] maintains a bibliography, currently with more than 280 entries, for work on distinguishing sets.

2. Existence result

Clearly if $N(u) = N(v)$, then $N(u) \cap S = N(v) \cap S$ for every $S \subseteq V(G)$, so G could not have an OLD-set.

Observation 2.1 (Seo and Slater [15]). A graph G has an OLD-set if and only if G has no isolated vertex and $N(u) \neq N(v)$ for all pairs u, v of distinct vertices.

A vertex of degree one is called an *endpoint*, and its neighbor is called a *support vertex*. A *strong support vertex* x has two distinct endpoints in $N(x)$. By Observation 2.1, if G has a strong support vertex, then G does not have an OLD-set.

Proposition 2.1 (Seo and Slater [16]). *For a tree T of order $n \geq 3$, T has an OLD-set if and only if T does not contain a strong support vertex.*

We can wonder if there is a forbidden induced subgraph characterization of graphs having OLD-sets. The answer is negative. Nor, in fact, is there a forbidden induced subgraph characterization of graphs which do not have OLD-sets.

Proposition 2.2. *There is no forbidden induced subgraph characterization of the class S1 of graphs having OLD-sets nor of the class S2 of graphs not having OLD-sets.*

Proof. For any given graph G, select a vertex $v \in V(G)$ and add two new vertices v' and v'' and edges vv' and vv'' to form a graph G^* of order $|V(G)| + 2$. Because $N(v') = N(v'') = \{v\}$, G^* does not have an OLD-set, and G is an induced subgraph of G^*. It follows that no forbidden subgraph characterization of the class $S2$ of graphs not having OLD-sets exists.

Given G, for each $v \in V(G)$ add two new vertices v' and v'' and edges vv', vv'' and $v'v''$ to form a graph $H = G \circ K_2$ (the corona of G with K_2) of order $3 \cdot |V(G)|$. Then H has an OLD-set $S = \{v', v'' | v \in V(G)\} = V(H) - V(G)$ with $|S| = 2 \cdot |V(G)|$, and G is an induced subgraph of H. Hence, there is no forbidden subgraph characterization of the class $S1$ of graphs having OLD-sets. □

It was shown in [15] that the order of a graph G having OLD-sets is bounded above by $2^{OLD(G)} - 1$. We prove next the following interpolation result for the open neighborhood locating-dominating number of a graph. (Related results appear in Theorem 3.2 and Corollary 3.1.)

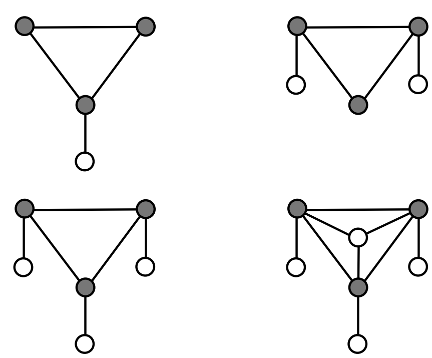


Figure 1. Graphs with $OLD(G) = 3$.

Proposition 2.3. *Assume (the number of detection devices available is) $k \geq 2$, and suppose $k+1 \leq n \leq 2^k - 1$, then there exists a connected graph G of order n with $OLD(G) = k$.*

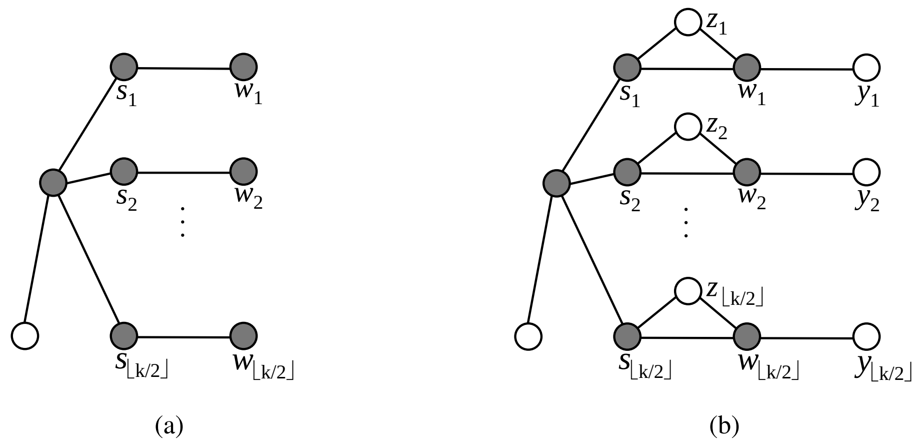


Figure 2. (a) Graph H and (b) Graph G with $OLD(H) = OLD(G) = k$.

Proof. For $k = 2$, $OLD(K_3) = 2$. For $k = 3$, there is a graph of order n, $4 \leq n \leq 7$ as shown in Figure 1.

Let k and n be integers with $k \geq 4$ and $k+1 \leq n \leq 2^k - 1$. Let us consider the following two situations.

Case 1: $k + 1 \leq n \leq 2k - 1$.

Let H be the graph obtained from a star $K_{1,\lceil k/2 \rceil}$ by subdividing exactly $\lfloor k/2 \rfloor$ edges as shown in Figure 2(a). Clearly, H has order $k+1$ and $OLD(H) = k$. We denote by $s_1, ..., s_{\lfloor k/2 \rfloor}$ the support vertices of degree two of H and by $w_1, ..., w_{\lfloor k/2 \rfloor}$ their leaf neighbors, respectively. If $n \leq \lceil 3k/2 \rceil$, then we add $n - (k+1)$ endpoints among $\{w_i : i = 1, 2, ..., \lceil k/2 \rceil - 1\}$ to obtain a graph G. One can easily check that G has order n and $OLD(G) = k$. Now suppose that $n > \lceil 3k/2 \rceil$. Let G be the graph obtained from H by adding each an endpoint y_i to each w_i except $w_{k/2}$ if k is even, and then we add $n - \lceil 3k/2 \rceil$ new vertices $z_1, z_2, .., z_{n - \lceil 3k/2 \rceil}$ by joining z_i to s_i and w_i, for $i = 1, 2, ..., n - \lceil 3k/2 \rceil$ as shown in Figure 2(b). Then G is a graph of order n and $OLD(G) = k$.

Case 2: $2k \leq n \leq 2^k - 1$.

Let G_k be the corona of a complete graph K_k and let $S = V(K_k)$. We note that S has $2^k - 1 - 2k$ distinct subsets of size i, where $2 \leq i \leq k$, and $i \neq k - 1$. For such subsets of S, select $n - 2k$ distinct subsets, and let G be the graph obtained from G_k by adding $n - 2k$ new vertices and attaching each new vertex to only one of these selected subsets so that no two new vertices are adjacent to the same subset. Then, it is easy to see that G has order n and $OLD(G) = |S| = k$. □

For the special case of trees we have the following. Note that for path P_4 we have $OLD(P_4)$=4.

Theorem 2.2 (Seo and Slater [16]). *If tree T of order $n \geq 5$ has an OLD-set, then $\lceil n/2 \rceil + 1 \leq OLD(T) \leq n - 1$.*

Theorem 2.3 (Seo and Slater [20]). *For $n \geq 5$ and $\lceil n/2 \rceil + 1 \leq j \leq n - 1$ there is a tree $T_{n,j}$ of order n with $OLD(T_{n,j}) = j$.*

As expected, determining the value of $OLD(G)$ is difficult, and the associated decision problem is NP-complete.

Open Neighborhood Locating-Dominating (OLD)

INSTANCE: Graph $G = (V, E)$ and positive integer $K \leq |V|$.

QUESTION: Is $OLD(G) \leq K$?

Theorem 2.4 (Seo and Slater [15]). *Problem OLD is NP-complete.*

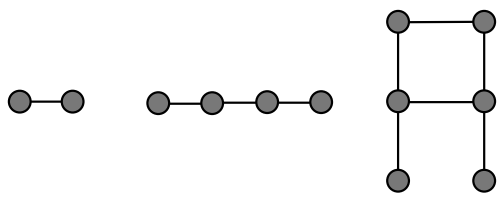


Figure 3. The three graphs G with $OLD(G) = |V(G)|$, P_2, P_4, and H.

3. Graphs G with small or large $OLD(G)$

Obviously, if a graph G has an OLD-set, then $2 \leq OLD(G) \leq n$. In this section, we characterize graphs G with $OLD(G) = 2, 3$ or n.

Observation 3.1. For a graph G, $OLD(G) = 2$ if and only if $G = K_2$ or K_3.

Proof. Let S be an OLD-set of G of size two. Then, clearly no vertex of S has external private neighbor in $V(G) - S$. Hence $|V(G) - S| \leq 1$ and the result follows. $\square$

Let $\mathcal{E}_1$ be the class of all graphs obtained from the corona of $K_3 \circ K_1$ by removing at most two leaves and possibly adding edges between remaining leaves. Let $\mathcal{E}_2$ be the class of all graphs obtained from a corona of $K_4 \circ K_1$ by removing at least one pendant edge and possibly adding edges between the remaining leaves.

Proposition 3.1. *For a graph G, $OLD(G) = 3$ if and only if $G \in \mathcal{E}_1 \cup \mathcal{E}_2$.*

Proof. First it is a routine matter to check that $OLD(G) = 3$ for every graph G in $\mathcal{E}_1 \cup \mathcal{E}_2$.

Now let G be a graph with $OLD(G) = 3$, and let S be a minimum cardinality OLD-set of G. The subgraph induced by S, denoted $G[S]$, must be complete because it can not contain an isolated vertex or be the path P_3. Thus $G[S] \cong K_3$. Also $V(G) - S \neq \emptyset$ for otherwise $G = K_3$ and by Observation 3.1, $OLD(K_3) = 2$. Now since any vertex of S has two neighbors in S, every vertex of $V(G) - S$ has either one or three neighbors in S. Moreover, there is at most one vertex in $V(G) - S$, say v^*, adjacent to all S. Note that $V(G) - S$ may contain adjacent vertices. Now it is straightforward to see that if v^* does not exist, then $G \in \mathcal{E}_1$, and if such a vertex v^* exists, then $G \in \mathcal{E}_2$. □

Let H be the graph of order 6 in Figure 3.

Theorem 3.2. *For a graph G of order n, $OLD(G) = n$ if and only if any component of G is a P_2, P_4 or H.*

Proof. Let G be a graph of order n and $OLD(G) = n$. Without loss of generality, we assume that G is connected. It is easy to see that if $n \leq 4$, then $G = P_2$ or P_4. Thus assume that $n \geq 5$. Clearly, $G \neq K_n$ since $OLD(K_n) = n - 1$. For a vertex $r \in V(G)$, let $V_r = V(G) - \{r\}$ and G_r the subgraph of G induced by V_r.

Let v be a vertex of minimum degree in G, $deg_G(v) = \delta(G)$. Note that G_v has no isolated vertices. Since V_v is not an OLD-set of G, there exist two non-adjacent vertices x and y, where $x \in N(v)$ and $y \notin N(v)$ such that $N(x) = N(y) \cup \{v\}$. Observe that G_y has no isolated vertices. Thus since V_y is not an OLD-set, there exist two non-adjacent vertices $z \in N(y)$ and $w \notin N(y)$ such that $N(z) = N(w) \cup \{y\}$. Because $y \in N(z)$, we have $x \in N(z)$ and $x \neq w$. We need to prove the following claims.

Claim 1: $w = v$.

Proof of Claim 1. Suppose to the contrary that $w \neq v$. Since $z \in N(y)$ and $N(x) = N(y) \cup \{v\}$, then $xz \in E(G)$. Also since $N(z) = N(w) \cup \{y\}$ we have $x \in N(w)$. But then $wy \in E$ since $N(x) = N(y) \cup \{v\}$, a contradiction.

Hence $w = v$ and so $N(z) = N(v) \cup \{y\}$.

Claim 2: $d_G(v) = 1$.

Proof of Claim 2. Assume that $d_G(v) \geq 2$. If $d_G(x) = 2$, then $d_G(y) = 1$, contradicting the fact that v is a vertex of minimum degree. Thus $d_G(x) \geq 3$, and so $d_G(y) \geq 2$. Because $\delta(G) \geq 2$, G_z has no isolated vertices. Since V_z is not an OLD-set of G, there exist two non-adjacent vertices $a \in N(z)$ and $b \notin N(z)$ such that $N(a) = N(b) \cup \{z\}$. Let us suppose that $a \neq y$. Then $a \in N(v)$, since $N(z) = N(v) \cup \{y\}$. Now since $N(a) = N(b) \cup \{z\}$, we have $b \in N(v)$ and so $b \in N(z)$, a contradiction. Hence $a = y$, implying that $N(y) = N(b) \cup \{z\}$. Recall that $N(x) = N(y) \cup \{v\}$ and $d_G(y) \geq 2$. Since G_b has no isolated vertices, V_b is not an OLD-set and so there are two non-adjacent vertices $\alpha \in N(b)$ and $\beta \notin N(b)$ such that $N(\alpha) = N(\beta) \cup \{b\}$. Since $\alpha \in N(b)$, it is clear that α is adjacent to y and x, that is $x, y \in N(\alpha)$. It follows that $x, y \in N(\beta)$ since $N(\alpha) = N(\beta) \cup \{b\}$. Hence $\beta \in N(x) \cap N(y)$, implying that $\beta \in N(b)$, a contradiction. We deduce that $d_G(v) = 1$.

Claim 3: $d_G(y) = 2$.

Proof of Claim 3. By Claim 2, $d_G(v) = 1$, and since $N(z) = N(v) \cup \{y\}$, we obtain that $d_G(z) = 2$. If $d_G(y) = 1$, then G is path of order $n = 4$, a contradiction. Thus $d_G(y) \geq 2$, and so $d_G(x) \geq 3$. Assume now that $d_G(y) \geq 3$. Since $N(x) = N(y) \cup \{v\}$, it follows that every vertex in $N(y)$ has degree at least two. Since $N(z) = \{x, y\}$, we have that G_z has no isolated vertices and so V_z is not an OLD-set of G. Thus there exists a vertex b not adjacent to y and z such that $N(y) = N(b) \cup \{z\}$. Note that $|N(b)| \geq 2$ since $d_G(y) \geq 3$. Now G_b has no isolated vertices and V_b is not an OLD-set. Thus there exist two non-adjacent vertices $\alpha \in N(b)$ and $\beta \notin N(b)$ such that $N(\alpha) = N(\beta) \cup \{b\}$. By using an argument similar to the one that is used in the proof of Claim 2, we arrive at $\beta \in N(b)$, which is a contradiction.

We deduce that $d_G(y) = 2$.

Now by Claims 2 and 3 we have $d_G(v) = 1$, $d_G(y) = 2$. It follows that $d_G(x) = 3$. Let y' be the second neighbor of y. Clearly $d_G(y') \geq 2$ since $y' \in N(x)$. Since G_z has no isolated vertices and V_z is not an OLD-set of G, there is a vertex u such that $N(y) = N(u) \cup \{z\}$. It is clear that such a vertex u is adjacent to y' and so has degree one. Consequently, $G = H$.

The converse is easy to show. □

Using Proposition 2.3 and Theorem 3.2 we have the following.

Corollary 3.1. *If $k \geq 7$, then there exists a graph $G_{n,k}$ of order n with $OLD(G_{n,k}) = k$ if and only if $k + 1 \leq n \leq 2^k - 1$.*

A *cycle* on n vertices is denoted by C_n. Next, we characterize graphs G of order n with minimum degree $\delta(G) \geq 2$ and C_4-free having $OLD(G) = n - 1$. Note that C_4-free graphs are the graphs with no cycle C_4 (not necessarily induced).

Theorem 3.3. *Let G be a connected graph of order n with minimum degree $\delta(G) \geq 2$ that is C_4-free. Then $OLD(G) = n - 1$ if and only if G is the cycle C_5 or the graph $K_1 + t \cdot P_2$ (illustrated in Figure 5a).*

Proof. Clearly $OLD(C_5) = 4$. For $G = K_1 + t \cdot P_2$ as in Figure 5a, either $V(G) - z_1$ or $V(G) - z_2$ is an $OLD(G)$-set, and $OLD(K_1 + t \cdot P_2) = 2t = n - 1$.

Note that $G - z_2$ is the even ordered graph in Figure 5a with $OLD(G - z_2) = n - 2$, but $OLD(G - z_1) = OLD(t \cdot P_2) = 2t = |V(G - z_1)|$. For the converse it will be shown that every C_4-free graph G of order n with no C_4 contains a vertex z such that $OLD(G - z) = n - 1 = |V(G - z)|$.

Letting G be C_4-free with minimum degree $\delta(G) \geq 2$ and G of order n, assume $OLD(G) = n - 1$. Assume vertex x is such that $V(G) - x$ is an $OLD(G)$-set. Note that $V(G) - x$ is also an OLD-set for $G - x$, so $OLD(G - x)$ is defined. Assume that $OLD(G - x) \leq n - 2$, and let D be an $OLD(G - x)$-set.

Suppose first that $|D| \leq n - 3$. Because D is an $OLD(G - x)$-set but not an OLD-set for G, there is a vertex x_1 such that $N(x) \cap D = N(x_1) \cap D$. If $N(x) \subseteq D$, then, because $d_G(x) \geq 2$, x_1 and x have two common neighbors and G contains a 4-cycle, a contradiction.

Thus, there is a vertex $w \in N(x)$ with $w \notin D$. Consider $D \cup \{w\}$. If $y \in V(G-x)$ and $N(y) \cap (D \cup \{w\}) = N(x) \cap (D \cup \{w\})$, then $N(y) \cap D \neq \emptyset$ implies that y and x have two common neighbors, implying that we have a C_4, a contradiction. Any two vertices in $G-x$ are distinguished by D, so $D \cup \{w\}$ is an OLD-set of G with $|D \cup \{w\}| < n-1$, a contradiction.

Suppose that $|D| = n-2$. Let $V(G) - D = \{x, a\}$. Because D is an $OLD(G-x)$-set, but D is not an OLD-set for G, there is a vertex y such that $N(x) \cap D = N(y) \cap D$. If $|N(x) \cap D| \geq 2$, then x and y have two common neighbors and there is a C_4, a contradiction. Thus $d_G(x) = 2$ and $N(x) = \{a, b\}$ where $b \in D$. Because $ay \in E(G)$ implies we have a 4-cycle x, a, y, b, we have $ay \notin E(G)$. But now if $y \neq a$, then $d_G(y) \geq 2$ implies that y and x have two common neighbors in D, a contradiction. Thus $y = a$ and $N(a) = \{x, b\}$. If $d_G(b) = 2$, then $n = 3$ and $G = K_1 + P_2$. We can assume $d_G(b) > 2$. We claim that $V(G) - b = D^*$ is also an $OLD(G)$-set. Clearly a is the only vertex w with $N(w) \cap D^* = \{x\}$, x is the only vertex w with $N(w) \cap D^* = \{a\}$, and b is the only vertex w with $N(w) \cap D^*$ containing $\{a, x\}$. Suppose y_1 and y_2 have $N(y_1) \cap D^* = N(y_2) \cap D^*$. Because $N(y_1) \cap D \neq N(y_2) \cap D$, exactly one of y_1 and y_2 is adjacent to b, say it is y_1. Now $d_G(y_2) \geq 2$ implies that y_1 and y_2 have two common neighbors and there is a C_4, a contradiction.

Because $d_G(b) \geq 3$ and $V(G) - b$ is an $OLD(G)$-set, the above argument shows that $OLD(G - b) = n-1 = |V(G) - b|$. By Theorem 3.2 each component of the C_4-free graph $G - b$ is P_2 or P_4, and $\delta(G) \geq 2$ implies that $N(b)$ contains all endpoints in each component of $G - b$. Because G is C_4-free, no vertex of degree two in a P_4-component of $G - b$ is adjacent to b. If some component of $G - b$ is a P_4, let a_1 and a_2 be the endpoints of the P_4. Then $G - a_1 - a_2$ is an OLD-set of G unless $n = 5$ and $G = C_5$, completing the proof. □

Corollary 3.2. *If G is a connected graph of order n with $\delta(G) \geq 2$, girth $g(G) \geq 5$, and $OLD(G) = n-1$, then G is C_5.*

Figure 4 shows all trees of order n with $OLD(G) = n-1$ and Figure 5 shows some infinite families of graphs with $OLD(G) = n-1$.

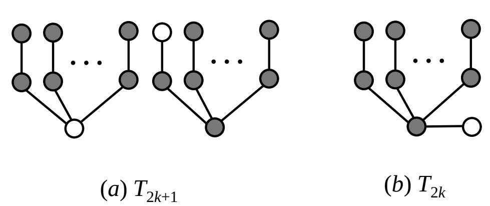
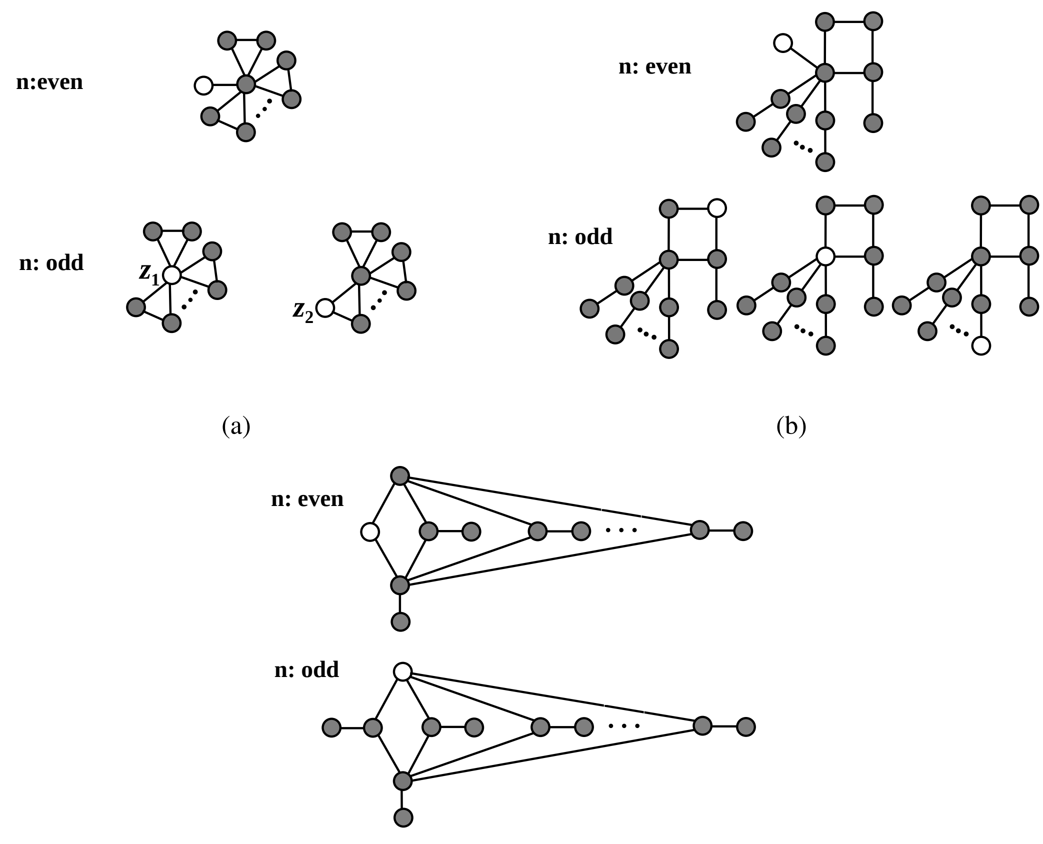


Figure 4. $OLD(T_n)$-sets where $OLD(T_n) = n-1$.



(c)

Figure 5. Infinite families of graphs with $OLD(G) = n - 1$.

4. Bounds

We first relate $OLD(G)$ to the packing number $\rho(G)$ [5], the maximum number of vertices which are pairwise at distance at least three.

Theorem 4.1. *Let G be a connected graph with minimum degree $\delta(G) \geq 3$ and C_4-free. Then $OLD(G) \leq n - \rho(G)$.*

Proof. Let P be a maximum packing set of G. Observe that since every vertex not in P has at most one neighbor in P, the subgraph induced by $V(G) - P$ has minimum degree at least two. Thus the subgraph induced by $V(G) - P$ has no isolated vertices. Now assume that $S = V(G) - P$ is not an OLD-set of G. Hence there are two vertices x, y such that $N(x) \cap S = N(y) \cap S$. Since $N[u] \cap N[v] = \emptyset$ for every pair $u, v \in P$, at most one of x and y belongs to P, implying that $xy \notin E$. Each of x and y has at least two neighbors in S. But then the subgraph induced by $N[x] \cup N[y]$ contains a cycle C_4 (not necessarily induced), a contradiction. Therefore S is an OLD-set for G implying that $OLD(G) \leq |V(G) - P| = n - \rho(G)$. □

As in Theorem 12 of Seo and Slater [15], using a "share"argument we obtain the next result.

Theorem 4.2 (Seo and Slater [15])**.** *For a graph G of order n and maximum degree $\Delta(G)$, If G has an OLD-set, then $OLD(G) \geq \frac{2n}{1+\Delta}$.*

Note that if G is a cubic graph, then $OLD(G) \geq n/2$.

Henning and Yeo [9] investigated the problem of determining the upper bound on $OLD(G)$ for a cubic graph G. To that end, they use an interplay between distinguishing-transversals in hypergraphs and identifying open codes in graphs. They first showed that identifying open codes in graphs can be translated to the problem of finding distinguishing-transversal in hypergraphs, and then using the result on the distinguishing-transversal problem they showed the following result.

Theorem 4.3 (Henning and Yeo [9])**.** *If G is a cubic graph of order n, then $OLD(G) \leq 3n/4$.*

We note that this bound is achieved, for example, by the cube Q_3 of order 8. To date, the largest known value of k for which there is an infinite family of cubic graphs G with $OLD(G) = k|V(G)|$ is $k = 3/5$. Henning and Yeo show such a family in [9].

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