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Negation switching invariant signed graphs

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Abstract

A *signed graph* (or, *sigraph* in short) is a graph G in which each edge x carries a value $\sigma(x) \in \{-,+\}$ called its *sign*. Given a sigraph S, the *negation* $\eta(S)$ of the sigraph S is a sigraph obtained from S by reversing the sign of every edge of S. Two sigraphs S_1 and S_2 on the same underlying graph are *switching equivalent* if it is possible to assign signs '+' ('plus') or '$-$' ('minus') to vertices of S_1 such that by reversing the sign of each of its edges that has received opposite signs at its ends, one obtains S_2. In this paper, we characterize sigraphs which are negation switching invariant and also see for what sigraphs, S and $\eta(S)$ are signed isomorphic.

Keywords: Balanced sigraph, Marked sigraph, Signed isomorphism, Switching equivalence
Mathematics Subject Classification : *Primary* ***05C 22****; Secondary 05C 75.*

1. Introduction

For standard terminology and notation in graph theory we refer to West (1996) and Zaslavsky (1998) for sigraphs. Throughout the paper, we consider finite, undirected graphs with no loops or multiple edges.

Formally, a *signed graph* (or, *sigraph* in short) is an ordered pair $S = (S^u, \sigma)$, where $S^u = (V, E)$ is a graph called the *underlying graph* of S and $\sigma : E \to \{+, -\}$ is a func-

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tion from the edge set E of S^u into the set $\{+,-\}$, called the *signature* (or $sign$ in short) of S. Alternatively, the sigraph can be written as $S=(V,E,\sigma)$, with V, E and σ in the above sense. Further, $E^+(S)$ will denote the set of all the edges of S^u that are mapped by σ to the element $+$ and $E^-(S)=E(S)-E^+(S)$. The elements of $E^+(S)$ are called *positive edges* and those of $E^-(S)$ are called *negative edges* of S. We may then regard any graph as a sigraph in which every edge is positive. In general, a *subsigraph* S' of S is said to be $all\ positive$ ($all\ negative$) if all the edges in S' are positive (negative). A subsigraph of S is said to be $homogeneous$ if it is either all-positive or all-negative and $heterogeneous$ otherwise. In a pictorial representation of a sigraph S, its positive edges are shown as bold line segments and negative edges as broken line segments. An example of such discrete structures is exhibited in Figure 1, where solid line segments represent edges that are assigned '+' and broken line segments represent those that are assigned '$-$'. Thus, in a pictorial representation of a graph, one would see only unbroken line segments alone, as every edge of a graph may be assumed to have been designated to be positive. By an $independent\ positive$ ($negative$) edge of S we mean a positive (negative) edge of S at each end of which no other positive (negative) edge of S is incident. The $negation$ $\eta(S)$ of the sigraph S is a sigraph obtained from S by reversing the sign of every edge of S. A sigraph S and its negation $\eta(S)$ are shown in Figure 1.

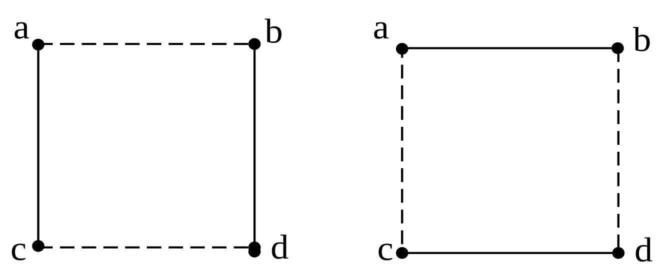

Figure 1. **A sigraph and its negation**

Sigraphs were introduced by Harary (1953) as prototype models to represent structures of cognitive inter relations between two individuals in a social group. Ever since, sigraphs have received much attention in social psychology because of their extensive use in modeling a verity of cognitive-based social processes ($e.g.$, see Abelson and Rosenberg, 1958; Harary, 1953,1957; Katai and Iwai, 1978**a**, 1978**b**; Fiksel, 1980; Acharya and Joshi, 2003; Kovchegov, 1994). Further intensive study of the topic has been due to their subsequently discovered connections with classical mathematical systems ($e.g.$, Zaslavsky, 1998; Singh, 2004; Acharya and Singh, 2004, 2005) used in solving a verity of problems of theoretical and practical interest.

Here $\langle V_i \rangle$ determines the *induced subsigraph* of S on the vertex subset V_i of $V(S)$ whereas $\langle V_i \rangle^u$ determines the underlying subgraph of S^u which is induced by the vertex subset V_i of $V(S^u)$. Subsigraphs may also be induced by sets of edges. If $S^{'}$ is the set of edges, the edge-induced subsigraph $\langle S^{'} \rangle$ is the subsigraph of S whose edge set is $S^{'}$ and whose vertex set consists of all ends of edges of $S^{'}$.

Two sigraphs S_1 and S_2 are *signed isomorphic* (written $S_1 \cong S_2$ or sometimes $S_1 = S_2$) if there exists an one-to-one correspondence between their vertex sets which preserve adjacency as well as sign.

Any function $\mu : V(S) \longrightarrow \{-, +\}$ is called a *marking* of the sigraph S and S_μ is then called the *marked sigraph*. $Switching$ a sigraph S with respect to the given marking μ means to obtain another sigraph $S = \mathbf{S}_\mu(S)$ from S_μ by changing the sign of each of its edge $x = uv$ for which $\mu(u) = -\mu(v)$ (as shown in Figure 2). The resulting sigraph $\mathbf{S}_\mu(S)$ is called $switched\ sigraph$. A sigraph S $switches$ to another sigraph S' written $S \sim S'$, whenever there exists a marking μ such that $S' \cong \mathbf{S}_\mu(S)$, where '$\cong$' denotes the usual equivalence relation of isomorphism in the class of sigraphs. It is obvious that '$\sim$' is an equivalence relation on the class of all sigraphs, and as such on the class $\Psi(G)$ of all sigraphs S such that their underlying graphs S^u are isomorphic to G. Hence, if $S \sim S'$ we shall say that S and S' are $switching\ equivalent$.

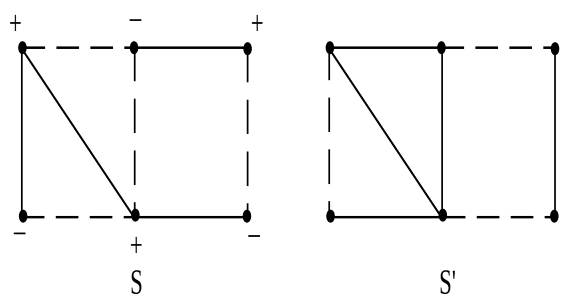

Figure 2. **Two sigraphs S and S' such that $S \sim S'$.**

Further, two cycles C_1 and C_2 are said to be *adjacent cycles* if and only if they have at least one vertex in common. A cycle that is an induced subgraph is called *chordless cycle*.

One can extend the study of a graph equations with respect to isomorphism to a sigraph equations with respect to switching equivalence (Gill and Patwardhan, 1981, 1986; Acharya, 1986). In this paper, we initiate study of a new system of switching equivalence relations, $i.e.$, negation switching invariant, aimed at hopefully facilitating application of results to analyze evolution of structures of social systems due to local interactions. We also obtain in the sequel conditions for which $S \cong \eta(S)$; infact, it is the subset of the set of solutions of $S \sim \eta(S)$.

2. Negation switching invariant sigraphs

Given a sigraph S and a positive integer n, the *n-path sigraph* $(S)_n$ is defined to be a sigraph on the vertex set $V(S)$ of S, with two vertices u and v joined by an edge $e = uv$ in $(S)_n$ provided

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there is a *u-v* path of length n in S and with the sign $\sigma_n(e)$ of e defined to be '$-$' if and only if in every *u-v* path of length n in S all the edges are negative (as shown in Figure 3). The notion of *n-path graphs* was introduced by Escalante et al. (1974), and various studies concerning this notion may be found in the work of Acharya (1973), Escalante and Montejano (1973, 1974), Harary et al. (1982), Simic (1983), etc. A graph G for which

$$(G)_n \cong G \tag{1}$$

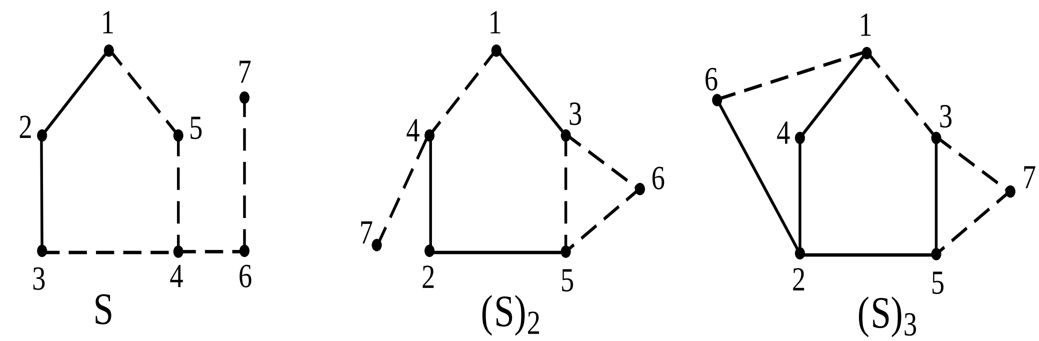

Figure 3. **Showing a sigraph S and its 2-path sigraph $(S)_2$ and 3-path sigraph $(S)_3$.**

has been termed as *n-path invariant graph* by Escalante and Montejano (1974), where the explicit solutions to (1) are determined for n = 2, 3. The structure of n-path invariant graphs remain still uninvestigated in literature for all $n \geq 4$. Clearly, $(S)_1$ = S for any sigraph S and hence to solve $S \sim \eta(S)$ is the special case for characterizing sigraphs S for $n = 1$ in $(S)_n \sim \eta(S)$. The following result has been already obtained.

Remark 2.1. A sigraph S = (G, σ) is switching equivalent to its negation $\eta(S)$ if it is a bipartite sigraph.

Towards this end, the following notion is needed: two sigraphs S_1 and S_2 are said to be *weakly isomorphic* (*e.g.*, *see* Sozański, 1980) or *cycle isomorphic* (*e.g.*, *see* Zaslavsky, 1982) if there exists an isomorphism $f : S_1^u \rightarrow S_2^u$ such that the sign of every cycle Z in S_1 equals the sign in S_2 (*i.e.*, f preserves both vertex adjacencies and the signs of the cycles of S_1 and S_2), where the sign of any subsigraph of a sigraph is defined as the product of the signs of its edges. The following theorem will also be useful in our further investigation, where $\Psi(G)$ will denote the set of all sigraphs whose underlying graph is G.

Theorem 2.1. (Sozański, 1980; Zaslavsky, 1982): *Given a graph G, any two sigraphs in $\Psi(G)$ are switching equivalent if and only if they are cycle isomorphic.*

The following theorem determines the solution to $S \sim \eta(S)$.

Theorem 2.2. *For a connected sigraph $S = (S^u, \sigma)$, $S \sim \eta(S)$ if and only if either*

(i) *S^u is bipartite or*

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(ii) *there exist subsets V_1 and V_2 of $V(S)$ such that*

- (a) $S = \langle V_1 \rangle \cup \langle V_2 \rangle$ *and $\langle V_1 \cap V_2 \rangle$ is bipartite,*
- (b) $\langle V_1 \rangle^u \cong \langle V_2 \rangle^u$ *such that degrees of corresponding vertices are preserved in S, and*
- (c) *each odd $($even$)$ cycle in $\langle V_1 \rangle$ is of opposite $($same$)$ sign to the corresponding cycle in $\langle V_2 \rangle$.*

Proof. **Necessity**: Let us suppose that $S \sim \eta(S)$. It is clear that $S^u \cong (\eta(S))^u$. Thus, there exists a bijection, say f, from the vertex set of S^u to the vertex set of $(\eta(S))^u$ $i.e.$,

$$f : V(S^u) \to V((\eta(S))^u)$$

such that $f(u) = u$ and $f(v) = v$ and two vertices u and v are adjacent in S^u if and only if $f(u)$ and $f(v)$ are adjacent in $(\eta(S))^u$. By Theorem 2, of cycle isomorphism, two sigraphs S_1 and S_2 are switching equivalent if and only if they are cycle isomorphic.

Now $S \sim \eta(S)$ implies that there exists a bijection ψ from the set of the cycles of S to the set of cycles of $\eta(S)$ such that cycles Z and $\psi(Z)$ are cycle isomorphic. Let $Z_1, Z_2, \ldots, Z_r$ be cycles in S which corresponds to $\psi(Z_1), \psi(Z_2), \ldots, \psi(Z_r)$, respectively in $\eta(S)$. Let $n_1, n_2, \ldots, n_r$ be the number of edges in $Z_1, Z_2, \ldots, Z_r$ respectively and $n'_1, n'_2, \ldots, n'_r$ be the number of negative edges in $Z_1, Z_2, \ldots, Z_r$ respectively. Since $S \sim \eta(S)$, we have

$$n'_1 + n'_2 + \cdots + n'_r + (n_1 - n'_1) + (n_2 - n'_2) + \cdots + (n_r - n'_r) \equiv 0 \pmod 2$$

$$n_1 + n_2 + \cdots + n_r \equiv 0 \pmod 2$$

This implies either all the cycles in S are of even length or odd cycles in S are even in number. If all the cycles are of even length then S^u is bipartite.

If S contains odd cycles also, then odd cycles are even in number. Now for every odd cycle Z_i in S, $Z_i \not\sim \eta(Z_i)$, but since S is cycle isomorphic to $\eta(S)$, implies that for each odd cycle Z_i in S there exists another odd cycle Z_i' in $\eta(S)$ such that $Z_i \sim Z_i'$ in such a manner that vertex adjacencies and sign of other cycles of S and $\eta(S)$ are preserved. This further implies that there exists another cycle Z_{ii} in S which is of the same length as Z_i but with opposite sign. Then, by the above argument, it is clear that there exists another bijection from the vertex set of S^u to the vertex set of $(\eta(S))^u$, which is different from f and the sigraphs S and $\eta(S)$ are switching equivalent with respect to this bijection. Let g be such a bijection $i.e.$,

$$g : V(S^u) \to V((\eta(S))^u)$$

Thus, there exist two vertices $u, v \in V(S^u)$ such that $g(u) \neq u$ and two vertices u and v are adjacent in S^u if and only if $g(u)$ and $g(v)$ are adjacent in $(\eta(S))^u$ and $S \sim \eta(S)$ with respect to this bijection.

Now, we first choose a cycle, say Z_1, which is odd and keep all the vertices of Z_1 in subset V_1 of $V(S)$. Then, since $Z_1 \not\sim \eta(Z_1)$ there exists another cycle Z_1' in $\eta(S)$ such that $Z_1 \sim Z_1'$ and vertex adjacencies and sign of other cycles of S and $\eta(S)$ are preserved. Put the vertices of Z_1', which are also the vertices of a cycle say Z_{11} in S, in subset V_2 of $V(S)$. Clearly $Z_1 \sim \eta(Z_{11})$.

Next, we choose another cycle, say Z_2, in S such that Z_2 is adjacent to Z_1 and Z_2 is different from Z_{11}. If no such Z_2 exists in S, then result is already proved. Next, if Z_2 exists then consider two cases:

Case I: If Z_2 is an odd cycle then by the above argument we can find a cycle Z_{22} in S such that $Z_2 \sim \eta(Z_{22})$ and vertex adjacencies and sign of other cycles of S and $\eta(S)$ are preserved and $\langle V(Z_1) \cup V(Z_2)\rangle \sim \langle V(\eta(Z_{11})) \cup V(\eta(Z_{22}))\rangle$. Put the vertices of cycle Z_2 in S in subset V_1 and vertices of cycle Z_{22} in S in subset V_2.

Case II: If Z_2 is an even cycle then again there exists a cycle Z_{22} in S such that $Z_2 \sim \eta(Z_{22})$ and $\langle V(Z_1) \cup V(Z_2)\rangle \sim \langle V(\eta(Z_{11})) \cup V(\eta(Z_{22}))\rangle$. Now Z_2 being an even cycle, the cycle Z_2 may be same as Z_{22} or it may be distinct.

If Z_2 and Z_{22} are not distinct, put the vertices of Z_2 or Z_{22} in both the vertex subsets V_1 and V_2. If Z_2 and Z_{22} are distinct cycles then put the vertices of Z_2 in V_1 and the vertices of Z_{22} in V_2.

Continuing in this manner we have a set of chordless cycles $Z_1, Z_2, \ldots, Z_r$ such that $\langle V(Z_1) \cup V(Z_2) \cup \cdots \cup V(Z_r)\rangle \sim \langle V(\eta(Z_{11})) \cup V(\eta(Z_{22})) \cup \cdots \cup V(\eta(Z_{rr}))\rangle$, where the vertices of half of the odd cycles in S are in V_1 and the vertices of other half odd cycles are in V_2. The remaining cycles, which are symmetric difference of these cycles, are automatically settled. Thus, by $S \sim \eta(S)$, it is clear that if $V(Z_i) \neq V(Z_i')$, then vertices of Z_i are put in V_1 and vertices of Z_i' are put in V_2 and if $V(Z_i) = V(Z_i')$, then vertices of $Z_i (= Z_i')$ are put in both the subsets V_1 and V_2, where $Z_i' = f(Z_i)$ and $Z_i' \in \eta(S)$. Since, $V(S) = V(\eta(S))$, the vertices of Z_i' are also in $V(S)$. Thus all the vertices, which are in any cycle of S, are in V_1 or in V_2. Now, we discuss only about those vertices which are not in any cycle of S.

Next, suppose $v \in V(S)$ is any vertex (if any) such that v dose not lie in any cycle of S, but adjacent with a vertex which lies on any cycle in S. If the vertex v is such that $g(v) = v$, then put the vertex v in both the subsets V_1 and V_2.

Now, suppose $v \in V(S)$ is such that $g(v) \neq v$. Then, consider two cases. If v is adjacent with a vertex which lies in V_1 (V_2), then put the vertex v in V_1 (V_2) and if v is adjacent with the vertex which is in both the subsets V_1 and V_2, then put v in V_1 and $g(v)$ in V_2 or vice-versa.

Next, we choose another vertex v_1 (if any), which is adjacent with v, such that v_1 is not in any cycle of S. Again, if $g(v_1) = v_1$, then put v_1 in both the subsets V_1 and V_2 and if $g(v_1) \neq v_1$, then if v is in V_1 (V_2) put v_1 in V_1 (V_2). Continuing in this manner we put all the vertices in V_1 or in V_2. Then, V_1 and V_2 are subsets of $V(S)$ such that $\langle V_1\rangle^u \cong \langle V_2\rangle^u$ and $S = \langle V_1\rangle \cup \langle V_2\rangle$. Clearly,

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if $V(Z_i) = V(Z_i')$, then Z_i cannot be an odd cycle. Thus, there can not be an odd cycle whose vertices are in V_1 and V_2 both and hence $\langle V_1 \cap V_2 \rangle$ is bipartite. Thus, conditions are necessary.

Sufficiency: If S^u is bipartite then it is trivially true that $S \sim \eta(S)$. On the other hand if S^u is not bipartite then it contains at least one odd cycle. By $(ii)(b)$ there exist two subsets V_1 and V_2 in S such that $\langle V_1 \rangle^u \cong \langle V_2 \rangle^u$. Since, $S^u \cong (\eta(S))^u$ and $V(S) = V(\eta(S))$, the same subsets V_1 and V_2 exist in $\eta(S)$ also. Also, by the same condition, degrees of corresponding vertices are preserved in $\eta(S)$ also. It is clear that there exist at least two isomorphism. One where every element is mapped to itself and another where element of $V_1 - V_2$ of S gets mapped to the element of $V_2 - V_1$ of $\eta(S)$ and the element of $V_2 - V_1$ of S gets mapped to the element of $V_1 - V_2$ of $\eta(S)$. Thus, there exists a bijection from the vertex set of S^u to the vertex set of $(\eta(S))^u$ such that every element in S^u is not an image of itself in $(\eta(S))^u$. If there is an odd (even) cycle in $\langle V_1 \rangle$ ($\langle V_2 \rangle$) in S, then by condition $(ii)(c)$, there is an odd (even) cycle in $\langle V_2 \rangle$ ($\langle V_1 \rangle$) in S of same length but of opposite (same) sign. By our bijection there is an odd (even) cycle of same length and same sign in $\langle V_2 \rangle$ ($\langle V_1 \rangle$) of $\eta(S)$. Thus, S and $\eta(S)$ are cycle isomorphic. Hence, by Theorem 2.1, $S \sim \eta(S)$. □

Lemma 2.1. *For a disconnected sigraph $S = (S^u, \sigma)$, $S \sim \eta(S)$ if and only if either*

(i) *S^u is bipartite or*

(ii) *there exist subsets V_1 and V_2 of $V(S)$ such that*

- (a) *$S = \langle V_1 \rangle \cup \langle V_2 \rangle$ and $\langle V_1 \cap V_2 \rangle$ is bipartite,*
- (b) *$\langle V_1 \rangle^u \cong \langle V_2 \rangle^u$ such that degrees of corresponding vertices are preserved in S, and*
- (c) *each odd (even) cycle in $\langle V_1 \rangle$ is of opposite (same) sign to the corresponding cycle in $\langle V_2 \rangle$.*

The result immediately follows from Theorem 2.2.

The following theorem determines the solution to $S \cong \eta(S)$.

Theorem 2.3. *For a given sigraph $S = (S^u, \sigma)$, $S \cong \eta(S)$ if and only if the edge set $E(S)$ can be partitioned into two subsets E_1 and E_2 such that $\langle E_2 \rangle \cong \langle \eta(E_1) \rangle$ and degrees of corresponding vertices are preserved in S.*

Proof. **Necessity**: Let $S \cong \eta(S)$. We know that the two sigraphs are signed isomorphic if and only if there exists an one-to-one correspondence between the vertices of the two sigraphs such that adjacencies along with signs of the two sigraphs are preserved. Let f be a bijection from the vertex set $V(S)$ to the vertex set $V(\eta(S))$ $i.e.,$

$$f : V(S) \to V(\eta(S))$$

such that any two vertices u and v of S are adjacent if and only if $f(u)$ and $f(v)$ are adjacent in $\eta(S)$ and are of the same signature. Let

$$\sigma' : E(\eta(S)) \longrightarrow \{-, +\}$$

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Then

$$\sigma(uv) = \sigma^{'}(f(u)f(v))$$

If $f(u) = u_1$ and $f(v) = v_1$ such that $u, v \in V(S)$, then $f(u) \neq u$ and $f(v) \neq v$, since otherwise f will not be signed isomorphism. Hence there exist $u_1, v_1 \in V(\eta(S))$ such that u corresponds to u_1 and v corresponds to v_1 under f and if $uv \in E(S)$ then $f(u)f(v) = u_1v_1 \in E(\eta(s))$. Also, $S \cong \eta(S)$ implies that $S^u \cong \eta(S)^u$, so there exists a bijection

$$g : V(S^u) \longrightarrow V(\eta(S)^u)$$

such that $g(u) = u$ and $g(v) = v$ but $\sigma(uv) = -\sigma^{'}(g(u)g(v)) = -\sigma^{'}(uv)$.

There also exists another isomorphism $g^{'}$ from vertex set of S^u to the vertex set of $\eta(S)^u$ which also preserve adjacencies $i.e.$,

$$g^{'} : V(S^u) \longrightarrow V(\eta(S)^u)$$

such that $g^{'}(u) = u^{'}$ and $g^{'}(v) = v^{'}$ and any two vertices u and v are adjacent in S if and only if u' and v' are adjacent in $\eta(S)$ and $\sigma(uv) = \sigma^{'}(u^{'}v^{'})$ where $u, v \in V(S)$ and $u^{'}, v^{'} \in V(\eta(S))$.

Under suitable choice of $u^{'}$ and $v^{'}$

$$\sigma^{'}(g^{'}(u)g^{'}(v))g^{'}(u)g^{'}(v) = f(u)f(v)$$

and $\sigma^{'}(u_1v_1) = \sigma^{'}(u^{'}v^{'})$.

Now, we first choose an edge, say $e_1 = uv$, and keep this edge in subsets E_1 of $E(S)$. Since $S \cong \eta(S)$ and f is a bijection from the vertex set $V(S)$ to the vertex set $V(\eta(S))$ such that $f(u) = u_1$ and $f(v) = v_1$, so by the definition of isomorphism $u_1v_1 = e_1^{'}$ is an edge in $\eta(S)$ and $\sigma(uv) = \sigma^{'}(u_1v_1)$. If u_1v_1 is an edge in $\eta(S)$ then, since the vertices of S and $\eta(S)$ are same and $S^u \cong \eta(S)^u$, $u_1v_1 = e_{11}$ is an edge in S and $\sigma^{'}(u_1v_1) = -\sigma(u_1v_1)$. So put the edge $u_1v_1 = e_{11}$ of S in subset E_2 of $E(S)$. It is easy to see that u corresponds to u_1, v corresponds to v_1 in S and $\sigma(uv) = -\sigma(u_1v_1)$ such that $u, v, u_1, v_1 \in V(S)$.

Next, we choose another edge, say e_2, in S such that it is adjacent to uv and it is different from u_1v_1. Now we can find an edge in S, say e_{22}, such that $\langle e_1 \cup e_2 \rangle \cong \langle \eta(e_{11}) \cup \eta(e_{22}) \rangle$. Every time we choose an edge in S which is neither selected in subset E_1 nor in E_2 of $E(S)$.

Continuing in this manner we have a set of edges $e_1, e_2, \ldots, e_r$ and $e_{11}, e_{22}, \ldots, e_{rr}$ in S such that $\langle e_1 \cup e_2 \cup \cdots \cup e_r \rangle \cong \langle \eta(e_{11}) \cup \eta(e_{22}) \cup \cdots \cup \eta(e_{rr}) \rangle$. Thus we can partition the edge set $E(S)$ into two subsets $E_1 = \langle e_1 \cup e_2 \cup \cdots \cup e_r \rangle$ and $E_2 = \langle \eta(e_{11}) \cup \eta(e_{22}) \cup \cdots \cup \eta(e_{rr}) \rangle$ such that $\langle E_2 \rangle \cong \langle \eta(E_1) \rangle$ and it is also clear that the degrees of corresponding vertices are same in S.

Sufficiency: Now, let we assume that we can partition the edge set $E(S)$ into two subsets E_1 and E_2 such that $\langle E_2 \rangle \cong \langle \eta(E_1) \rangle$ and the degrees of corresponding vertices are same in S.

Since, $\langle E_2 \rangle \cong \langle \eta(E_1) \rangle$ thus there exists an one-to-one correspondence between $\langle E_2 \rangle$ and $\langle \eta(E_1) \rangle$. Since subsigraph $\langle E_1 \rangle$ is isomorphic to $\langle \eta(E_1) \rangle$ or $\langle E_2 \rangle$ and $\langle E_2 \rangle$ is isomorphic to $\langle \eta(E_2) \rangle$ or $\langle E_1 \rangle$ in $\eta(S)$ such that adjacencies are also preserve and degrees of corresponding vertices are same in S. So there exists an isomorphism from the vertex set of $V(S)$ to the vertex set of $V(\eta(S))$. Hence, $S \cong \eta(S)$, which completes the proof.

□

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